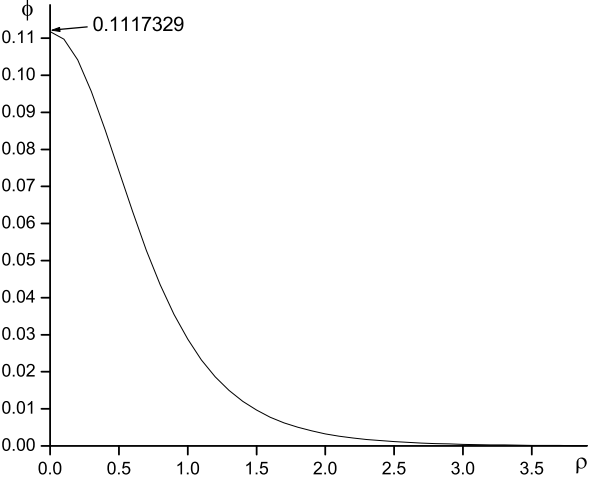
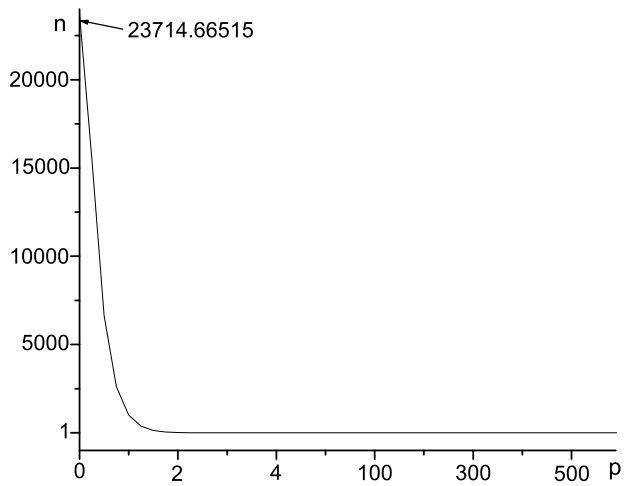


## Figure Captions

Figure 1: The thick-walled bubble profile.

Figure 2: The spectrum of created particles.





# Particle production by the thick-walled bubble

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## Abstract

The spectrum of created particles during the tunneling process, leading to the decay of a false vacuum state, is studied numerically in the thick-wall approximation. It is shown that in this case the particle production is very intensive for small momenta. The number of created particles is nearly constant  $n(p) \approx 1$  for  $4 \leq p \leq 500$ .

## 1 Introduction

In the semiclassical approximation, false vacuum decay through the barrier penetration is described by the nontrivial  $O(4)$  symmetric solution of Euclidean (imaginary-time) equation of motion with the boundary condition that the field asymptotically approaches its false vacuum value, [1]. This solution is referred to as the bounce one. The bounce looks like a four-dimensional spherical bubble of true vacuum embedded in a false vacuum background. The bubble evolution after nucleation is obtained by the analytic continuation of the bounce solution to the Minkowskian time,  $\tau \rightarrow it$ . There exist two limiting cases, referred to as the thin and thick wall approximations respectively, when the theory of bubble formation is greatly simplified, [1, 2]. Thin wall approximation is realized when the energy-density difference between the true and false vacuum is much smaller than the height of the barrier. The particle production in this case has been studied in [3]. The analysis presented in that paper shows that the particle production is strongly suppressed in the thin-wall approximation. It is of interest to explicate whether this is the general feature for the created particles spectrum during the false vacuum tunneling when the background space-time curvature is neglected (without gravity). Throughout this paper we shall consider another limiting case, when the energy-density difference between the true and false vacuum is much greater than the height of the barrier. The paper is organized as follows. In Sec.2 we briefly describe the technique for calculation of the particle spectrum. In Sec.3, based on the calculational procedure described in Sec.2, we evaluate the particle spectrum for the thick-walled bubble. Sec.4 is devoted to the summary and concluding remarks.

## 2 Technique of calculation

For the calculation of particle spectrum we use the standard technique developed in [4, 5]. Throughout this paper the metric signature in Minkowski space-time is  $(-, +, +, +)$  and units  $\hbar = c = 1$  are used. We consider a scalar field  $\phi$  defined by the Lagrange density

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi)^2 - U(\phi), \quad (1)$$

with the potential

$$U(\phi) = \frac{\lambda}{2}\phi^2(\phi - 2a)^2 - \epsilon(\phi^3/2a^3 - 3\phi^4/16a^4), \quad (2)$$

where  $\lambda$ ,  $a$  and  $\epsilon$  are positive parameters. The potential (2) has a local minimum at  $\phi_f = 0$ ,  $U(0) = 0$ , a global minimum at  $\phi_t = 2a$ ,  $U(2a) = -\epsilon$ , and a local maximum at  $\phi_{top} = 8\lambda a^5 / (8\lambda a^4 + 3\epsilon)$ ,  $U(\phi_{top}) = 128\lambda^3 a^{12} (2\lambda a^4 + \epsilon) / (8\lambda a^4 + 3\epsilon)^3$ . The  $O(4)$  symmetric bounce solution  $\phi_b$  satisfies the Euclidean equation of motion

$$\frac{d^2\phi}{d\rho^2} + \frac{3}{\rho} \frac{d\phi}{d\rho} = U'(\phi), \quad (3)$$

with the boundary conditions  $\phi(\infty) = 0$ ,  $d\phi/d\rho = 0$  at  $\rho = 0$ , where  $\rho = \sqrt{\vec{x}^2 + \tau^2}$  and the prime denotes differentiation with respect to  $\phi$ . Since the bounce solution is  $O(3, 1)$  invariant in the Minkowskian space-time and  $O(4)$  invariant in the Euclidean region it is convenient to use the following coordinate systems. The coordinate system used in the Euclidean region is  $(\rho, \chi, \theta, \varphi)$  where  $(\theta, \varphi)$  are usual angle coordinates on two-dimensional sphere and  $(\rho, \chi)$  are related to  $r = |\vec{x}|$  and  $\tau$  as follows

$$\begin{aligned} r &= \rho \sin(\chi), \quad \tau = -\rho \cos(\chi), \\ 0 &\leq \chi \leq \pi/2, \quad 0 \leq \rho < \infty. \end{aligned} \quad (4)$$

The coordinate system in the Minkowskian region may be obtained by replacement  $(\rho, \chi) \rightarrow (-i\rho_M, -i\chi_M)$ , which yields

$$\begin{aligned} r &= \rho_M \sinh(\chi_M), \quad t = \rho_M \cosh(\chi_M), \\ 0 &< \chi_M < \infty, \quad 0 < \rho_M < \infty. \end{aligned} \quad (5)$$

The equation governing the fluctuation field  $\Phi$  reads

$$\left[ -\partial_M^2 - \frac{3}{\rho_M} \partial_M + \frac{1}{\rho_M^2} \hat{L}^2 - U''(\phi_b) \right] \Phi = 0, \quad (6)$$

where  $\partial_M$  denotes the partial derivative with respect to  $\rho_M$  and  $\hat{L}$  is the Laplacian operator on three-dimensional unit hyperboloid. Expanding  $\Phi$  in terms of harmonic functions on the three-dimensional hyperboloid  $Y_{plm}$ ,  $-\hat{L}^2 Y_{plm} = (1 + p^2) Y_{plm}$ , the Eq.(6) for the mode function takes the form

$$\left[ \partial_M^2 + \frac{(p^2 + 1/4)}{\rho_M^2} + U''(\phi_b) \right] \psi = 0. \quad (7)$$

Making use of analytic continuation of  $\psi$  to the Euclidean region by replacement  $\rho_M \rightarrow i\rho$  one obtains the equation for the mode function in the under-barrier region

$$\left[ \partial_\rho^2 + \frac{(p^2 + 1/4)}{\rho^2} - U''(\phi_b) \right] \psi_E = 0, \quad (8)$$

where  $\psi_E$  is connected with  $\psi$  by the asymptotic boundary condition  $\psi_E(-i\rho_M) = \psi(\rho_M)$  at  $\rho_M \rightarrow 0$ . At  $\tau = -\infty$  the field  $\phi$  is in false vacuum state and correspondingly the fluctuation field satisfies the vanishing boundary condition  $\psi_E \rightarrow 0$  when  $\rho \rightarrow \infty$ . Since  $U''(\phi_b) \rightarrow 4a^2\lambda$  when  $\rho \rightarrow \infty$  the asymptotic solution of Eq.(7) will have the form

$$\psi(\rho_M) = c_1(p) \sqrt{\rho_M} H_{ip}^{(1)}(2a\sqrt{\lambda}\rho_M) + c_2(p) \sqrt{\rho_M} H_{ip}^{(2)}(2a\sqrt{\lambda}\rho_M), \quad (9)$$

where  $H_{ip}^{(1)}(x)$  and  $H_{ip}^{(2)}(x)$  are the Hankel functions of the first and second kinds, respectively. The spectrum of created particles  $n(p)$  is given by [4, 5]

$$n(p) = \frac{e^{-2p\pi}}{||c_1(p)/c_2(p)||^2 - e^{-2p\pi}}. \quad (10)$$

For detailed description of the general formalism see [4, 5].

### 3 Particle spectrum

For the thick-wall case we specify the potential parameters as follows  $a = 1$ ,  $\lambda = 0.5$ ,  $\epsilon = 100$ . For that values of parameters the ratio  $U(\phi_{top})/\epsilon = 0.5752022890e - 6$ . The corresponding bounce solution obtained by the program code odex with local tolerance  $1.0D - 17$  is shown in Fig.1. The detailed description of this program code is given in [6]. Taking into account that  $\phi_b(10) = 0.3453646895630613e - 08$  and using the program package Maple 7 one finds that the solution of Eq.(8) for  $\rho \geq 10$  satisfying the vanishing boundary condition when  $\rho \rightarrow \infty$  is given by

$$\psi_E = a_3 \sqrt{\rho} K_{ip}(\sqrt{2}\rho), \quad (11)$$

where  $K_{ip}(x)$  is the modified Bessel function of the second kind. In order to make the analytic continuation at  $\rho = 0$  we construct the analytic solution to Eq.(8) in the vicinity of  $\rho = 0$ . For this purpose, the power series expansion of  $\phi_b$  about  $\rho = 0$  is used for evaluating of  $U''(\phi_b)$

$$U''(\phi_b) = -29.34386061 + 50.39180321\rho^2 - 52.71178464\rho^4 - 21.76409860\rho^6 + O(\rho^8). \quad (12)$$

Since we want to use the expansion (12) for evaluating the solution of Eq.(8) in the region  $0 < \rho < 0.001$  we omit the terms higher than quadratic. Inserting this expression into Eq.(8) and using the program package Maple 7 one obtains the following solution

$$\psi_E = \frac{a_1}{\sqrt{\rho}} M(1.033421066, 0.5 i p, 7.09871842\rho^2) + \frac{a_2}{\sqrt{\rho}} W(1.033421066, 0.5 i p, 7.09871842\rho^2), \quad (13)$$

where  $M$ ,  $W$  denote the Whittaker functions. For  $\rho_M \geq 10$  the solution of Eq.(7) is given by

$$\psi(\rho_M) = c_1(p) \sqrt{\rho_M} H_{ip}^{(1)}(\sqrt{2}\rho_M) + c_2(p) \sqrt{\rho_M} H_{ip}^{(2)}(\sqrt{2}\rho_M). \quad (14)$$

Since the particle number depends on the ratio  $|c_1(p)/c_2(p)|$  the multiplier  $a_3$  in Eq.(11) may be fixed arbitrarily. The Eq.(11) and its derivative give the initial conditions to Eq.(8) at  $\rho = 10$ . Using these initial values the solution of Eq.(8) in the region  $0.001 \leq \rho < 10$  is constructed numerically by the code odex with local tolerance  $1.0D - 17$ . Matching this solution with Eq.(13) at  $\rho = 0.001$  one finds the coefficients  $a_1$ ,  $a_2$ . Making use of analytic continuation  $\rho \rightarrow -i\rho$  the Eq.(13) now gives the initial conditions to Eq.(7) at  $\rho_M = 0.001$ . With these initial conditions the Eq.(7) is solved again numerically up to  $\rho_M = 10$  by the odex with local tolerance  $1.0D - 17$ . Matching this solution with Eq.(14) and using Eq.(10) one evaluates the particle number for given  $p$ . For evaluating of special functions and their derivatives at the matching points we have used the program package Maple 7. The spectrum obtained in this way is shown in Fig.2. As  $p$  increases from 0 to 4 the number of created particles decreases from 23714.66515 to 1.000916176 and becomes nearly constant  $n(p) \approx 1$  for  $4 \leq p \leq 500$ . For the sake of clarity the behavior of  $n(p)$  for small values of momentum is shown in the following table.

p	0.25	0.5	0.75	1	1.25
n	15524.29139	6622.761821	2612.820881	1008.019999	378.9347624
p	1.5	1.75	2	2.25	2.5
n	138.1763281	49.01973850	17.20855685	6.281386004	2.664255183

### 4 Summary

Using the standard technique [4, 5] we have shown that, in contrast to the thin-wall case, the particle production in the thick-wall approximation is very intensive. The number of created particles is

especially large at small values of momentum, decreases sufficiently fast as  $p$  increases from 0 to 4 and becomes nearly constant  $n(p) \approx 1$  for  $4 \leq p \leq 500$ . To date we have been unable to evaluate the behavior of  $n(p)$  for asymptotically large values of  $p$ . In the thin-wall case the particle production is strongly suppressed in general, it is nearly constant for small momenta and behaves as  $\exp(-2p\pi)$  for large values of momentum [3]. As we observe the particle production has a quite different nature in the thick-wall case. A few remarks are in order. This result is not strictly correct since we do not take into account the constraint on the fluctuation field that arises due to fact that the proper fluctuation field associated with the tunneling is the transverse part of total fluctuation field with respect to the bounce solution [7]. In [7] the application of this constraint is illustrated for the spatially homogenous tunneling. It was shown that, due to this constraint, the spatially homogenous tunneling does not allow the particle production with zero momentum. If such a restriction will not reduce the particle number significantly in the thick-wall case, one has to consider the back reaction of particle production on the tunneling. On the other hand the analysis of a concrete thick-wall model, when the quartic term is ignored in Eq.(2), shows that the activation rate at zero temperature makes a dominant contribution to the vacuum decay in comparison with the tunneling rate [8]. Unfortunately we do not know whether this is valid in the thick-wall approximation in general and if this is the case how does the zero temperature activation rate depend on  $a^4\lambda/\epsilon$ .

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## References

- [1] S.Coleman, Phys. Rev. **D15** (1977) 2929.
- [2] A.Linde, Nucl. Phys. **B216** (1983) 421.
- [3] M.Maziashvili, Mod. Phys. Lett. **A18** (2003) 993.
- [4] K.Yamamoto, T.Tanaka, and M.Sasaki, Phys. Rev. **D51** (1995) 2968;
- [5] T.Hamazaki, M.Sasaki T.Tanaka, and K.Yamamoto, Phys. Rev. **D53** (1996) 2045.
- [6] E.Hairer, S.Norsett and G.Wanner, Solving Ordinary Differential Equations, Vol.1 (Mir, Moscow, 1990).
- [7] M.Maziashvili, Mod. Phys. Lett. **A18** (2003) 1895.
- [8] E.Calzetta, A.Roura and E.Verdaguer, Phys. Rev. **D64** (2001) 105008.